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LETTER TO THE EDITOR

A photon rest mass and the dispersion of longitudinal electric waves in interstellar space

R BURMAN

Department of Physics, University of Western Australia, Nedlands, WA 6009, Australia

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Abstract. This letter treats the propagation of longitudinal electric waves in a cold plasma, which can occur if the photon has a nonzero rest mass.

A number of authors has considered the dispersion arising from a nonzero photon rest mass m of transverse electromagnetic waves (eg Gintsburg 1964, Goldhaber and Nieto 1971, Kobzarev and Okun' 1968); the possibility of detection of such dispersion in radiation of cosmic origin, in the presence of dispersion arising from the interstellar (Lee 1971) or intergalactic (Burman 1972) plasma, has been discussed. In a cold plasma, as in free space, longitudinal electric waves can propagate if $m \neq 0$; the dispersion of such waves will be discussed here.

Let $c \equiv (\mu_0\epsilon_0)^{-1/2}$, μ_0 and ϵ_0 being the permeability and permittivity of free space, and write m as $\hbar\omega_c/c^2$, $\hbar$ being Planck's constant divided by 2π. In MKS units, Proca's equations lead to the wave equations

$$(\Box^2 - \omega_c^2/c^2)\phi = -\rho/\epsilon_0 \tag{1}$$

and

$$(\Box^2 - \omega_c^2/c^2)A = -\mu_0 J \tag{2}$$

for the scalar and vector potentials ϕ and A; the net charge and current densities are denoted by ρ and J.

Suppose that the fields are small amplitude disturbances in a plasma which is stationary in the unperturbed state and has unperturbed electron number density N, corresponding to an angular plasma frequency ω_p; ion motion will be neglected. Let e and m_e denote the charge and mass of an electron; v will denote the electron fluid velocity, and a time factor $e^{i\omega t}$ will be taken. In the linear approximation, $J = Nev$ and, from conservation of charge, $\rho = (ie/\omega)\nabla . (Nv)$. Suppose that the plasma is homogeneous, so that ω_p is constant, and isotropic: there will be no coupling between longitudinal and transverse waves; the former have an electric field E, but no magnetic field.

If n denotes the refractive index, (1) and (2) give

$$\left(1 - n^2 - \frac{\omega_c^2}{\omega^2}\right)\phi = -cn\frac{m_e}{e}\frac{\omega_p^2}{\omega^2}v_l \tag{3}$$

and

$$\left(1 - n^2 - \frac{\omega_c^2}{\omega^2}\right)A = -\frac{m_e}{e}\frac{\omega_p^2}{\omega^2}v \tag{4}$$

where v_l is the longitudinal component of v. Since $E = -\nabla\phi - \partial A/\partial t$, it follows from

(3) and (4) that

$$E = \frac{\mathrm{i}\omega m_\mathrm{e}}{e}\frac{\omega_\mathrm{p}^2}{\omega^2}\frac{v - n^2 v_\mathrm{l}}{1 - n^2 - (\omega_\mathrm{c}^2/\omega^2)}. \tag{5}$$

If the plasma is cold, then the linearized equation of motion for the electron fluid is $\mathrm{i}\omega v = eE/m_\mathrm{e}$. Hence, from (5), for longitudinal waves

$$n^2 = \frac{1 - (\omega_\mathrm{p}^2 + \omega_\mathrm{c}^2)/\omega^2}{1 - (\omega_\mathrm{p}^2/\omega^2)}. \tag{6}$$

The refractive index is zero when $\omega = \omega_\infty$, where $\omega_\infty \equiv (\omega_\mathrm{p}^2 + \omega_\mathrm{c}^2)^{1/2}$, and is infinite when $\omega = \omega_\mathrm{p}$. In an actual plasma, thermal effects and damping will prevent the refractive index from becoming infinite.

The dispersion relation (6) can also be written

$$\left(1 - \frac{\omega_\mathrm{p}^2}{\omega^2}\right)(n^2 - 1) = -\frac{\omega_\mathrm{c}^2}{\omega^2}. \tag{7}$$

When $\omega_\mathrm{c} = 0$, this has a spurious root $n^2 = 1$, together with the root $\omega^2 = \omega_\mathrm{p}^2$ which represents the usual, nonpropagating, plasma oscillation.

If $\omega^2 \gg \omega_\mathrm{p}^2$, then (6) reduces to the free-space dispersion relation, namely

$$n^2 = 1 - \omega_\mathrm{c}^2/\omega^2. \tag{8}$$

If $\omega^2 \ll \omega_\mathrm{p}^2$, then (6) gives

$$n^2 \simeq 1 + \omega_\mathrm{c}^2/\omega_\mathrm{p}^2 \tag{9}$$

and there is no dispersion. At sufficiently low frequencies, ion motion should be allowed for.

It is known (Goldhaber and Nieto 1971) that $m < 4 \times 10^{-48}$g, corresponding to $\omega_\mathrm{c}/2\pi < \frac{1}{2}\,\mathrm{s}^{-1}$. In the interstellar plasma, $\omega_\mathrm{p}/2\pi \simeq 10^3\ \mathrm{s}^{-1}$, so that $\omega_\mathrm{p}^2 \gg \omega_\mathrm{c}^2$. For frequencies well above $10^3\,\mathrm{s}^{-1}$, (8) holds and dispersion depends on ω_c only. For frequencies well below $10^3\,\mathrm{s}^{-1}$, but exceeding a few tens of s^{-1}, (9) holds: there is little dispersion and n is near one.

If they exist at all, longitudinal electric waves in free space or a cold plasma will be only weakly coupled to matter (Bass and Schrödinger 1955), which will make them very difficult to detect. If they do exist, such waves could travel across cosmic distances for frequencies much lower than in the case of transverse waves: the latter are cut-off for $\omega < \omega_\infty$, whereas the former are cut-off for $\omega_\mathrm{p} < \omega < \omega_\infty$. It has been suggested (Gertsenshtein 1971) that the events detected by Weber might be caused by longitudinal electric waves.

Further investigations of longitudinal electric waves, in intergalactic as well as interstellar space, are in progress.

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